PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Specialist TEST 4 Weds 28 Aug 2019 TIME: 50 minutes working Classpads allowed No notes allowed 45 marks 8 Questions

Name:_______________________________

Teacher: ____________________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (3, 3 & 3 = 9 marks)
Determine the following integrals using the given substitutions.

a) $\int 3x\left(5x^2+1\right)^7 dx \qquad u = 5x^2+1$

b) $\int (5x-2)\sqrt{2x-1}\,dx \qquad u = 2x-1$

c) $\int \sec^2 x \tan^8 x\, dx \qquad u = \tan x$

Q2 (3 marks)

Identical twins Sherry and Mary were both given the following integral to solve. $\int 2\sin x\cos x\,dx$

Sherry's solution was as follows.

$\int 2\sin x\cos x\,dx \quad u=\sin x$

$\int 2u\cos x\dfrac{du}{\cos x}$

$\int 2u\,du = u^2 = \sin^2 x$

While Mary's solution was to:

$\int 2\sin x\cos x\,dx = \int \sin 2x\,dx = -\dfrac{1}{2}\cos 2x$

Explain why the solutions differ and state which is the correct answer. Show your reasoning.

Q3 (3 & 4 = 7 marks)

Determine the following integrals showing all working.

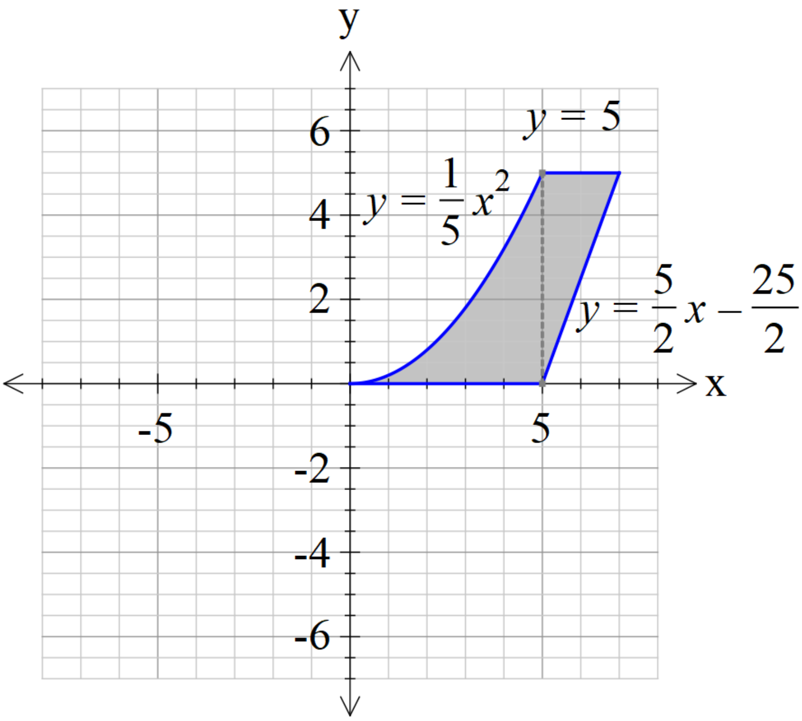
a) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x+\sin x}{\cos x-\sin x}\,dx$

Q3 cont-

b) $\int \frac{6x^3 + 11x^2 + 15x + 20}{(x+1)^2 (x^2 + 4)} dx$ (4 marks)

(Hint- set up simultaneous equations to solve for constants on your classpad)

Q4 (4 marks)
The shaded region is rotated about the y axis. Determine the volume of the resulting solid.



Q5 (1 & 4 = 5 marks)

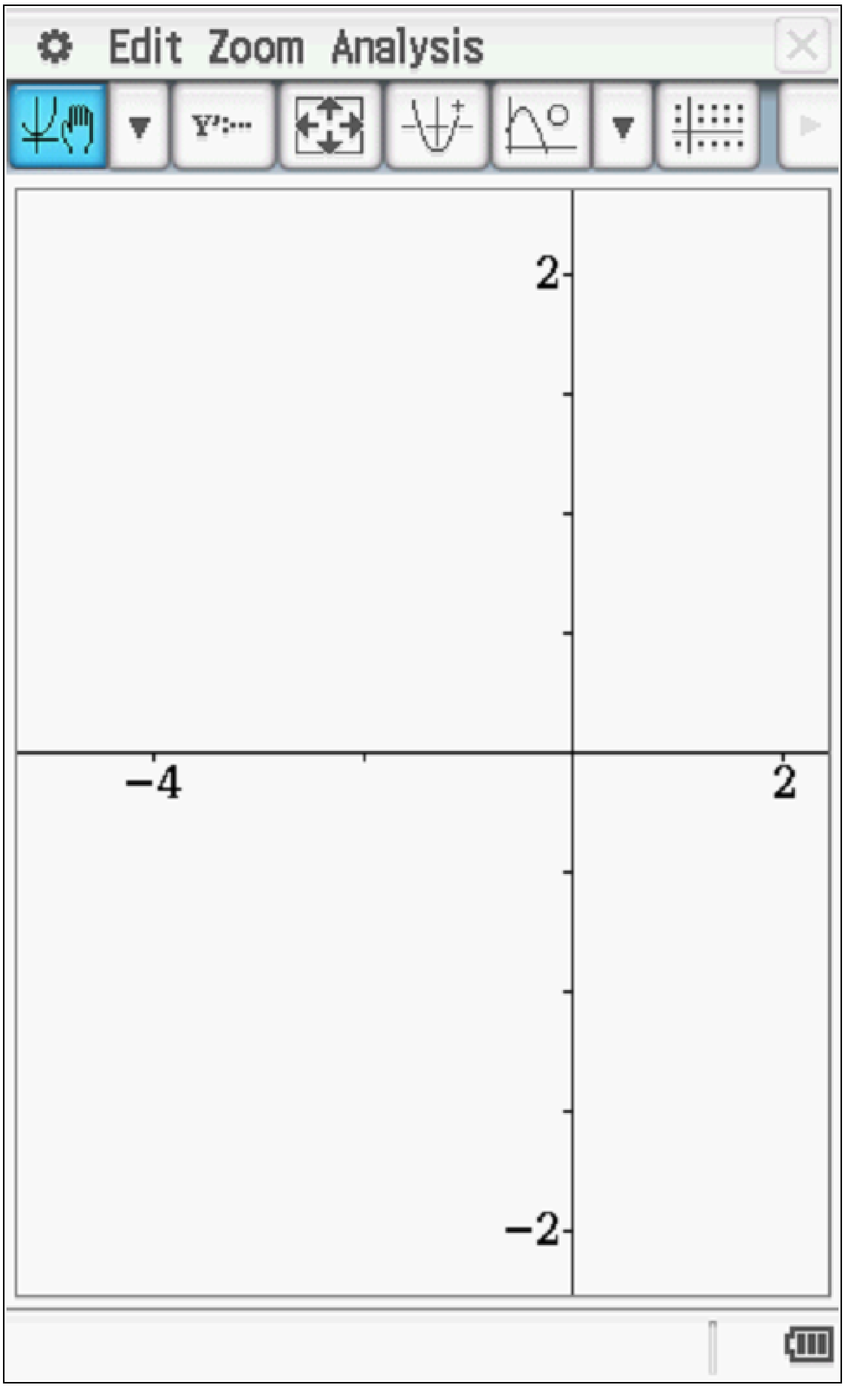
The mass, N grams, of a gas produced in a factory at time t seconds can be modelled by the logistical formula $\frac{dN}{dt} = 9N - 5N^2$ with an initial mass of 0.1 grams.

a) Determine the limiting mass as $t \to \infty$.

b) Show that $N = \frac{9}{5 + ce^{-9t}}$ and determine the constant.

Q6 (3 & 3 = 6 marks)

a) Sketch the slope field for $\frac{dy}{dx} = (1 - x)(x + 3)$ on the axes below.



b) Given that point A (-1,1) is a known point on our solution, show this curve on the slope field above and give the equation.

Q7 (2, 3 & 2 = 7 marks)

A particle with displacement, x metres from the origin at time t seconds, moves such that $x = 5\sin\left(2t + \frac{\pi}{3}\right)$.

a) Show that the motion is simple harmonic.

b) Determine the first two times that the speed is exactly half of the maximum speed.

c) Determine the distance travelled in the first 3 seconds.

Q8 (4 marks)
A particle with displacement, x metres from the origin at time t seconds, has an acceleration given by $\ddot{x} = -n^2 x$. The amplitude of the motion is given by A metres.

Show by integration that the speed, v metres per second, is given by $v^2 = n^2\left(A^2 - x^2\right)$.